

EN530.678 Nonlinear Control and Planning in Robotics
Homework #6
 March 30, 2022

Due: April 6, 2022 (before class)

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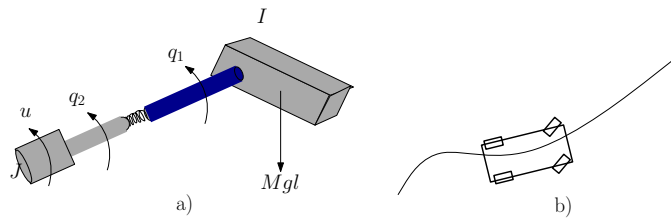


Figure 1: a) Flexible-joint robot b) Car-like robot

1. (15 pts) (Khalil) Consider the system

$$\dot{x}_1 = -x_1 + x_2 - x_3, \quad \dot{x}_2 = -x_1 x_3 - x_2 + u, \quad \dot{x}_3 = -x_1 + u, \quad y = x_3$$

- (a) (5 pts) Is the system input-output linearizable?
- (b) (5 pts) If yes, transform it into the normal form and specify the region over which the transformation is valid
- (c) (5 pts) Is the system minimum phase?

2. (15 pts) Consider the system

$$\dot{x}_1 = -x_2 - \frac{3}{2}x_1^2 - \frac{1}{2}x_1^3, \quad \dot{x}_2 = u$$

- (a) (10 pts) Design a globally stabilizing state feedback controller by using feedback linearization.
- (b) (5 pts) Implement the controller in Matlab. Starting from $(x_1, x_2) = (-4, 0)$, simulate the closed-loop dynamics and plot graphs of the states and control versus time.

3. (15 pts) Consider the dynamics of the flexible joint robot (Figure 1a) given by

$$I\ddot{q}_1 + Mgl \sin q_1 + k(q_1 - q_2) = 0$$

$$J\ddot{q}_2 + k(q_2 - q_1) = u.$$

The goal is to achieve a desired motion of the first joint q_1 expressed using the output function

$$y = q_1.$$

- (a) (10 pts) Show that the virtual input that renders this system feedback linearizable is

$$v = y^{(4)}$$

and provide the static feedback transformation

$$u = \alpha(x) + \beta(x)v$$

- (b) (5 pts) Find a control law for the virtual input v so that the error state given by

$$z = \begin{pmatrix} y - y_d \\ \dot{y} - \dot{y}_d \\ \ddot{y} - \ddot{y}_d \\ y^{(3)} - y_d^{(3)} \end{pmatrix}$$

stabilizes to zero. Note: it is enough to express the closed-loop system as $\dot{z} = Az$ and assume standard stability conditions on the chosen gains. Extra credit (3 pts): what are the exact algebraic stability requirements on the chosen gains?

4. (20 pts) Consider the control of a car-like robot (Figure 1b) with equations of motion

$$\begin{aligned} \dot{x} &= v \cos \theta \\ \dot{y} &= v \sin \theta \\ \dot{\theta} &= \frac{\tan u_1}{\ell} v \\ \dot{v} &= u_2 \end{aligned}$$

where the state (x, y, θ, v) denotes the the position, orientation, and forward velocity. The inputs u_1 and u_2 denote the steering wheels angle and the forward acceleration, respectively. The constant $\ell > 0$ denotes the distance between the axles.

- (a) (10 pts) Using feedback linearization, design a control law that can track a given trajectory $(x(t), y(t))$.
- (b) Implementation. Write a Matlab script `car_traj_f1.m` which implements your feedback-linearizing controller to follow a chosen reference trajectory. A file `uni_flat_f1.m` which demonstrates these functions for a related system is provided as a reference which you can use for your own implementation if you choose to.
- (4 pts) Plot the desired reference path
 - (6 pts) Implement your tracking controller and follow the generated path. Start at a “perturbed” initial state and show that your controller stabilizes to the desired trajectory.

Note: Note: Upload your code and plots as a .zip file (LastName.FirstName.HW6.zip) using <https://forms.gle/wDMCsuRfvczPGNDi6>. **In addition**, attach a printout of the code and all plots to your homework solutions.