EN530.678 Nonlinear Control and Planning in Robotics
Homework #3

February 25, 2022

Due: March 4, 2022 (before class)

Prof: Marin Kobilarov

1. **[Total 15 pts]** Let M be the ellipsoidal shell in $\mathbb{R}^3$ given by $x^2 + y^2 + 4z^2 = 1$. Show that M is a manifold.

2. **[Total 10 pts]** Let g_1 and g_2 denote vector fields on $\mathbb{R}^3$ (with coordinates (x, y, z)) defined by

$$g_1 = \begin{pmatrix} 0 \\ -z \\ y \end{pmatrix}, \qquad g_2 = \begin{pmatrix} y \\ -x \\ 0 \end{pmatrix}$$

   (a) **[5 pts]** Show that g_1 and g_2 can actually be defined as vector fields on the standard two sphere S^2 of radius one.

   (b) **[5 pts]** Calculate the Lie bracket $[g_1, g_2]$.

3. **[Total 15 pts]** Consider the distribution on $\mathbb{R}^3$ that is given at the point $(x, y, z) \in \mathbb{R}^3$ by the set of vectors $(a, b, c) \in \mathbb{R}^3$ satisfying $6ax + 2by + 10cz = 0$.

   (a) **[10 pts]** Show that the distribution is integrable.

   (b) **[5 pts]** Find the corresponding integrable manifolds defined by this distribution.

4. **[Total 8 pts]** A dynamical system in $\mathbb{R}^4$ can be decribed by

$$\dot{q} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} u_1 + \begin{pmatrix} q_2 \\ 0 \\ 1 \\ 0 \end{pmatrix} u_2 + \begin{pmatrix} q_3 \\ 0 \\ 1 \\ 1 \end{pmatrix} u_3,$$

   with input u_1, u_2, and u_3. Show it is nonholonomic.

5. **[Total 10 pts]** (MLS 7.2) Show that the differential constraint in $\mathbb{R}^5$ given by

$$(0,\ 1,\ \rho \sin q_5,\ \rho \cos q_3,\ \cos q_5)^T \dot{q} = 0,$$

   for $q \in \mathbb{R}^5$ is nonholonomic.