EN530.678 Nonlinear Control and Planning in Robotics
Homework #8

April 4, 2018

Due: April 11, 2018 (before class)

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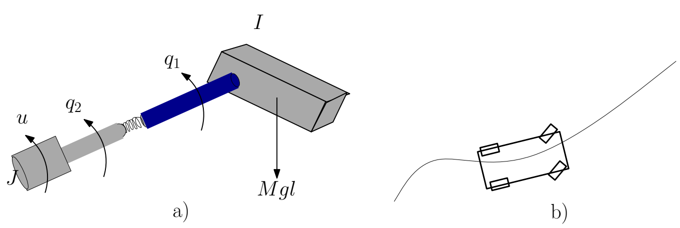

Figure 1: a) Flexible-joint robot b) Car-like robot

1. (15 pts) Consider the system

$$\dot{x}_1 = -x_2 - \frac{3}{2}x_1^2 - \frac{1}{2}x_1^3, \quad \dot{x}_2 = u$$

   (a) (10 pts) Using backstepping, design a controller which uses linear feedback in the first stage, i.e. the desired value of x_2 should be of the form kx_1 and choose k appropriately. In other words, we should have an error variable $z = x_2 - kx_1$ for the second stage.

   Note: a linear controller is preferred since it will result in smaller control effort when far away from the origin.

   (b) (5 pts) Compare the backstepping controller with the feedback linearization control law designed in the previous homework. Using Matlab simulations, compare their performance and the control effort used in each case starting from $(x_1, x_2) = (-4, 0)$. Plot representative graphs of the states and control against time.

   (c) (Optional: for extra credit (5 pts)*) Find a purely linear controller $u = k_1x_1 + k_2x_2$ that results in asymptotic stability?

2. (20 pts) Consider the control of a car-like robot (Figure 1b) with equations of motion

$$\dot{x} = v\cos\theta$$
$$\dot{y} = v\sin\theta$$
$$\dot{\theta} = \frac{\tan u_1}{\ell}v$$
$$\dot{v} = u_2$$

where the state (x, y, θ, v) denotes the the position, orientation, and forward velocity. The inputs u_1 and u_2 denote the steering wheels angle and the forward acceleration, respectively. The constant $\ell > 0$ denotes the distance between the axles.

(a) (10 pts) Using backstepping, design a control law that can track a given trajectory $(x(t), y(t))$.

(b) Implementation. Write a Matlab script `car_traj_bs.m` which implements your backstepping controller to follow a chosen reference trajectory. A file `uni_flat_bs.m` which demonstrates these functions for a related system is provided as a reference which you can use for your own implementation if you choose to.

   i. (4 pts) Plot the desired reference path

   ii. (6 pts) Implement your tracking controller and follow the generated path. Start at a "perturbed" initial state and show that your controller stabilizes to the desired trajectory. *Optional: inject noise in the controls along the path and observe performance.*

3. (10 pts) (Khalil) Consider the system

$$\dot{x}_1 = -x_1 + x_1 x_2, \quad \dot{x}_2 = x_2 + x_3, \quad \dot{x}_3 = x_1^2 + \delta(x) + u,$$

where $\delta(x)$ is an unknown (locally Lipschitz) function of x that satisfies $|\delta(x)| \leq k\|x\|$ for all x, with known constant k. Design a globally stabilizing state feedback controller.

4. (20 pts) Consider the dynamics of a mechanical system given by

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = u,$$

where the mass M, Coriolis C, and gravity g are functions that are not perfectly known. Their known nominal values are given by $\hat{M}, \hat{C}, \hat{g}$.

(a) (5 pts) Design a nominal controller $\hat{u} = \psi(x)$ which can asymptotically stabilize the nominal (known) system

$$\hat{M}(q)\ddot{q} + \hat{C}(q, \dot{q})\dot{q} + \hat{g}(q) = \hat{u},$$

to the origin $x = 0$, where we used the notation $x = (q, \dot{q})$.

(b) (5 pts) Express the original uncertain system according to

$$\dot{x} = f(x) + G(x)[u + \delta(x, u)],$$

where the functions $f(x)$ and $G(x)$ encode the known dynamics and $\delta(x, u)$ represents the uncertainty. Provide the exact expressions for f, G and δ.

(c) (5 pts) Design a robust controller $u = \psi(x) + v$ by providing a disturbance attenuation term v which renders the closed-loop system asymptotically stable outside of an infinitely-small region around the origin. You can assume that a bound on the uncertainty is provided in the form

$$\|\delta(x, \psi(x) + v)\| \leq \rho(\|x\|) + k_0\|v\|, \tag{1}$$

for some positive function ρ and a constant $0 \leq k_0 < 1$.

(d) (5 pts) A bound of the form (1) is possible only under certain conditions on the deviation of the terms M, C, g from their nominal values. What are these conditions?