EN530.678 Nonlinear Control and Planning in Robotics
Homework #7

March 28, 2018

Due: April 4, 2018 (before class)

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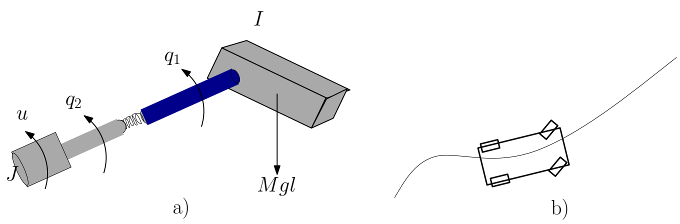

Figure 1: a) Flexible-joint robot b) Car-like robot

1. (15 pts) (Khalil) Consider the system

$$\dot{x}_1 = -x_1 + x_2 - x_3, \quad \dot{x}_2 = -x_1 x_3 - x_2 + u, \quad \dot{x}_3 = -x_1 + u, \quad y = x_3$$

   (a) (5 pts) Is the system input-output linearizable?

   (b) (5 pts) If yes, transform it into the normal form and specify the region over which the transformation is valid

   (c) (5 pts) Is the system minimum phase?

2. (15 pts) Consider the system

$$\dot{x}_1 = -x_2 - \frac{3}{2}x_1^2 - \frac{1}{2}x_1^3, \quad \dot{x}_2 = u$$

   (a) (10 pts) Design a globally stabilizing state feedback controller by using feedback linearization.

   (b) (5 pts) Implement the controller in Matlab. Starting from $(x_1, x_2) = (-4, 0)$, simulate the closed-loop dynamics and plot graphs of the states and control.

3. (15 pts) Consider the dynamics of the flexible joint robot (Figure 1a) given by

$$I\ddot{q}_1 + Mgl\sin q_1 + k(q_1 - q_2) = 0$$
$$J\ddot{q}_2 + k(q_2 - q_1) = u.$$

   The goal is to achieve a desired motion of the first joint q_1 expressed using the output function

$$y = q_1.$$

(a) (10 pts) Show that the virtual input that renders this system feedback linearizable is

$$v = y^{(4)}$$

and provide the static feedback transformation

$$u = \alpha(x) + \beta(x)v$$

(b) (5 pts) Find a control law for the virtual input v so that the error state given by

$$z = \begin{pmatrix} y - y_d \\ \dot{y} - \dot{y}_d \\ \ddot{y} - \ddot{y}_d \\ y^{(3)} - y_d^{(3)} \end{pmatrix}$$

stabilizes to zero. Note: it is enough to express the closed-loop system as $\dot{z} = Az$ and assume standard stability conditions on the chosen gains. Extra credit (5 pts): what are the exact algebraic stability requirements on the chosen gains?

4. (20 pts) Consider the control of a car-like robot (Figure 1b) with equations of motion

$$\begin{aligned} \dot{x} &= v \cos\theta \\ \dot{y} &= v \sin\theta \\ \dot{\theta} &= \frac{\tan u_1}{\ell} v \\ \dot{v} &= u_2 \end{aligned}$$

where the state (x, y, θ, v) denotes the the position, orientation, and forward velocity. The inputs u_1 and u_2 denote the steering wheels angle and the forward acceleration, respectively. The constant $\ell > 0$ denotes the distance between the axles.

(a) (10 pts) Using feedback linearization, design a control law that can track a given trajectory $(x(t), y(t))$.

(b) Implementation. Write a Matlab script `car_traj_fl.m` which implements your feedback-linearizing controller to follow a chosen reference trajectory. A file `uni_flat_fl.m` which demonstrates these functions for a related system is provided as a reference which you can use for your own implementation if you choose to.

i. (4 pts) Plot the desired reference path

ii. (6 pts) Implement your tracking controller and follow the generated path. Start at a "perturbed" initial state and show that your controller stabilizes to the desired trajectory. *Optional: inject noise in the controls along the path and observe performance.*