

# EN530.678 Nonlinear Control and Planning in Robotics

## Homework #5

March 7, 2018

Due: March 14, 2018 (before class)

Prof: Marin Kobilarov

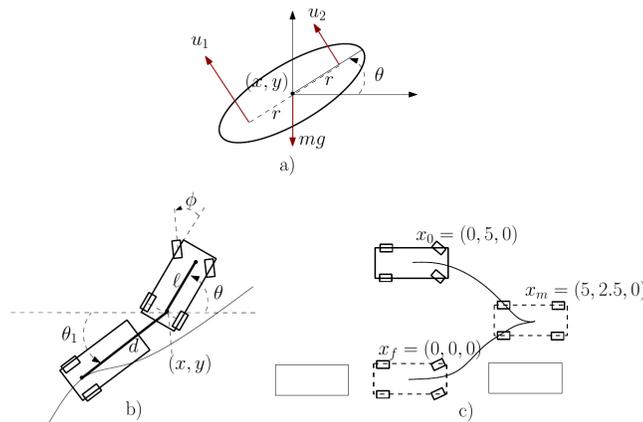


Figure 1: a) Planar body with two colinear forces, off center of mass b) Truck towing a trailer, c) Parallel parking for the car.

1. Read the paper “Differential Flatness of Mechanical Control Systems: A Catalog of Prototype Systems” (reference is also included in Lecture notes 7). The paper lists a system with *two colinear forces, off center* (Figure 1a). Write down the equations of motion of this system. Show that the system is differentially flat by expressing the whole state and inputs as functions of the flat outputs and their derivatives.
2. Consider the control of a truck with a trailer (Figure 1b) with equations of motion

$$\begin{aligned}\dot{x} &= \cos \theta u_1 \\ \dot{y} &= \sin \theta u_1 \\ \dot{\theta} &= \frac{\tan \phi}{\ell} u_1 \\ \dot{\phi} &= u_2 \\ \dot{\theta}_1 &= \frac{1}{d} \sin(\theta - \theta_1) u_1,\end{aligned}$$

where  $(x, y, \theta)$  are the position and orientation of the truck,  $\phi$  is the steering wheels angle,  $\theta_1$  is the angle of the trailer, and  $\ell$  and  $d$  are the lengths of the truck and the trailer.

Show that the system is differentially flat using the point between the rear wheels of the trailer as the flat output.

3. Consider a simplified kinematic model of a car-like robot with configuration  $x = (x_1, x_2, x_3)$ , where  $x_1, x_2$  denote the position and  $x_3$  the orientation. The vehicle is controlled with forward velocity  $u_1$  and steering angle  $u_2$ . The equations of motion  $\dot{x} = f(x, u)$  are given by

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} \cos x_3 u_1 \\ \sin x_3 u_1 \\ u_1 \frac{\tan u_2}{\ell} \end{pmatrix}$$

where the constant  $\ell > 0$  denotes the distance between the axles.

- (a) Give expressions for the state and controls in terms of the differentially flat outputs  $y = (x_1, x_2)$ .
  - (b) The car is required to perform a parallel parking maneuver starting at state  $x_0 = (0, 5, 0)$ , going forward to state  $x_m = (5, 2.5, 0)$  and backing-up to state  $x_f = (0, 0, 0)$ . Employing the fact the system is differentially flat, give explicit expressions for the trajectory  $x(t)$  and required inputs  $u(t)$  to generate the two segments of the parking maneuver. You can use polynomial interpolation or another basis function approach in flat output space. Assume that each segment takes time  $T = 10$  s. When designing the paths, you can assume that the magnitude of the initial and final velocity of the car in each segment can be chosen freely, e.g. you can set  $|u_1| = 1$  m/s. See Figure 1c.
4. Recall the two-link manipulator with dynamics given in Lecture Notes #2. The file `arm_test.m` implements the ODE and simulates the computed torque law for this system. Extend this code as follows:
    - (a) implement a trajectory generation routine using e.g. polynomial basis functions in flat outputs  $y(t) = q(t)$ . Denote the resulting trajectory by  $q_d(t)$  and the associated feedforward control by  $u_d(t)$ .

Note: Upload your code and plots as a .zip file using <https://goo.gl/forms/1jxhjYUI1KqYdi7g1>; in addition attach a printout of the code and all plots to your homework solutions.