EN530.678 Nonlinear Control and Planning in Robotics
Homework #3

February 21, 2018

Due: February 28, 2018 (before class)

Prof: Marin Kobilarov

1. Let M be the ellipsoidal shell in $\mathbb{R}^3$ given by $x^2 + y^2 + 4z^2 = 1$. Show that M is a manifold. (15 points)

2. Let X and Y define vector fields on $\mathbb{R}^3$ (with coordinates (x, y, z)) defined by

$$X = y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y}, \qquad Y = y\frac{\partial}{\partial x} - x\frac{\partial}{\partial y}$$

   (a) Show that X and Y can actually be defined as vector fields on the standard two sphere S^2 of radius one. (5 points)

   (b) Calculate the Lie bracket $[X, Y]$. (5 points)

3. Consider the distribution on $\mathbb{R}^3$ that is given at the point $(x, y, z) \in \mathbb{R}^3$ by the set of vectors $(a, b, c) \in \mathbb{R}^3$ satisfying $6ax + 2by + 10cz = 0$.

   (a) Show that the distribution is integrable. (10 points)

   (b) Find the corresponding integrable manifolds defined by this distribution. (5 points)

4. (MLS 7.2) Show that the differential constraint in $\mathbb{R}^5$ given by

$$(0,\ 1,\ \rho\sin q_5,\ \rho\cos q_3,\ \cos q_5)^T \dot{q} = 0,$$

   for $q \in \mathbb{R}^5$ is nonholonomic. (10 points)