EN530.678 Nonlinear Control and Planning in Robotics
Homework #1

February 7, 2018

Due: February 14, 2018 (before class)

Prof: Marin Kobilarov

1. (Khalil) For each of the following systems, use a quadratic Lyapunov function candidate to show that the origin is asymptotically stable:

   (a) $\dot{x}_1 = -x_1 + x_1 x_2, \qquad \dot{x}_2 = -x_2$

   (b) $\dot{x}_1 = -x_2 - x_1(1 - x_1^2 - x_2^2), \qquad \dot{x}_2 = x_1 - x_2(1 - x_1^2 - x_2^2)$

   (c) $\dot{x}_1 = x_2(1 - x_1^2), \qquad \dot{x}_2 = -(x_1 + x_2)(1 - x_1^2)$

   (d) $\dot{x}_1 = -x_1 - x_2, \qquad \dot{x}_2 = 2x_1 - x_2^3$

2. **Implementation:** For 1(a)-1(d), write a MATLAB script which plots $x(t)$ and your Lyapunov candidate $V(t)$ for $t \in [0, 5]$ starting from $x(0) = (0.5, 0.5)$. You can use the `ode45` function for integrating the dynamics.

   Note: Upload your code and plots as a .zip file using `https://goo.gl/forms/1jxhjYUIlKqYdi7g1`; in addition attach a printout of the code and all plots to your homework solutions.

3. (Khalil) Consider the system

   $$\dot{x}_1 = -x_2 x_3 + 1, \quad \dot{x}_2 = x_1 x_3 - x_2, \quad \dot{x}_3 = x_3^2(1 - x_3)$$

   (a) Show that the system has a unique equilibrium point.

   (b) Using linearization, show that the equilibrium point asymptotically stable. Is it globally asymptotically stable?

4. (Khalil) Consider the system

   $$\dot{x} = -a[I_n + S(x) + xx^T]x,$$

   where a is a positive constant, I_n is the nxn identity matrix, and $S(x)$ is an x-dependent skew symmetric matrix. Show that the origin is globally exponentially stable.